Electrical and Electronics Engineering
2024-2025
Master Semester 2

Course
Smart grids technologies

Time Dissemination Technologies for Synchrophasor Networks

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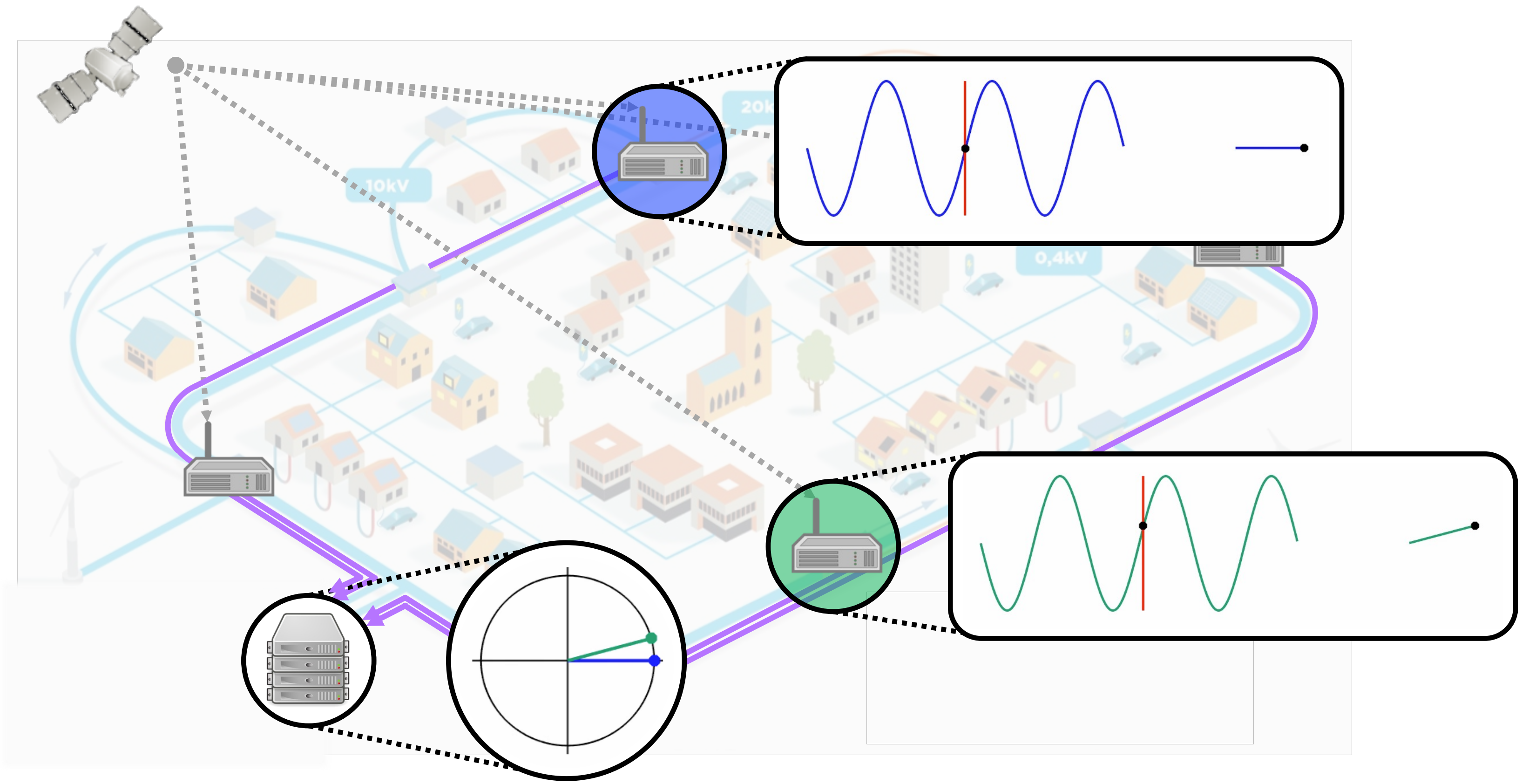
Context – Synchrophasor Networks



- PMU-B loses synchronized with UTC time
- At every time-stamp, PMU-B provides a wrong phase estimate
- The synchrophasors may lead to wrong control and protection actions at the PDC

Context – PMU timing accuracy

PMU accuracy metrics

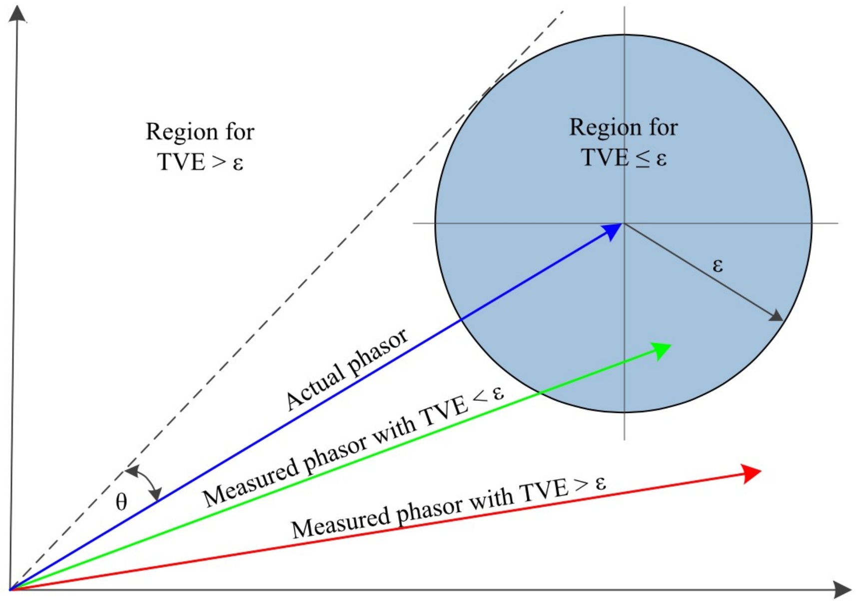
Frequency measurement Error $FE = \left| f_{true} - f_{measured} \right|$

ROCOF measurement Error $RFE = \left| (df/dt)_{true} - (df/dt)_{measured} \right|$

Total Vector Error $\text{TVE} = \sqrt{\dfrac{(\hat{X}_r - X_r)^2 + (\hat{X}_i - X_i)^2}{(X_r)^2 + (X_i)^2}}$

Estimated synchrophasor $\hat{X}_r + j\hat{X}_i$

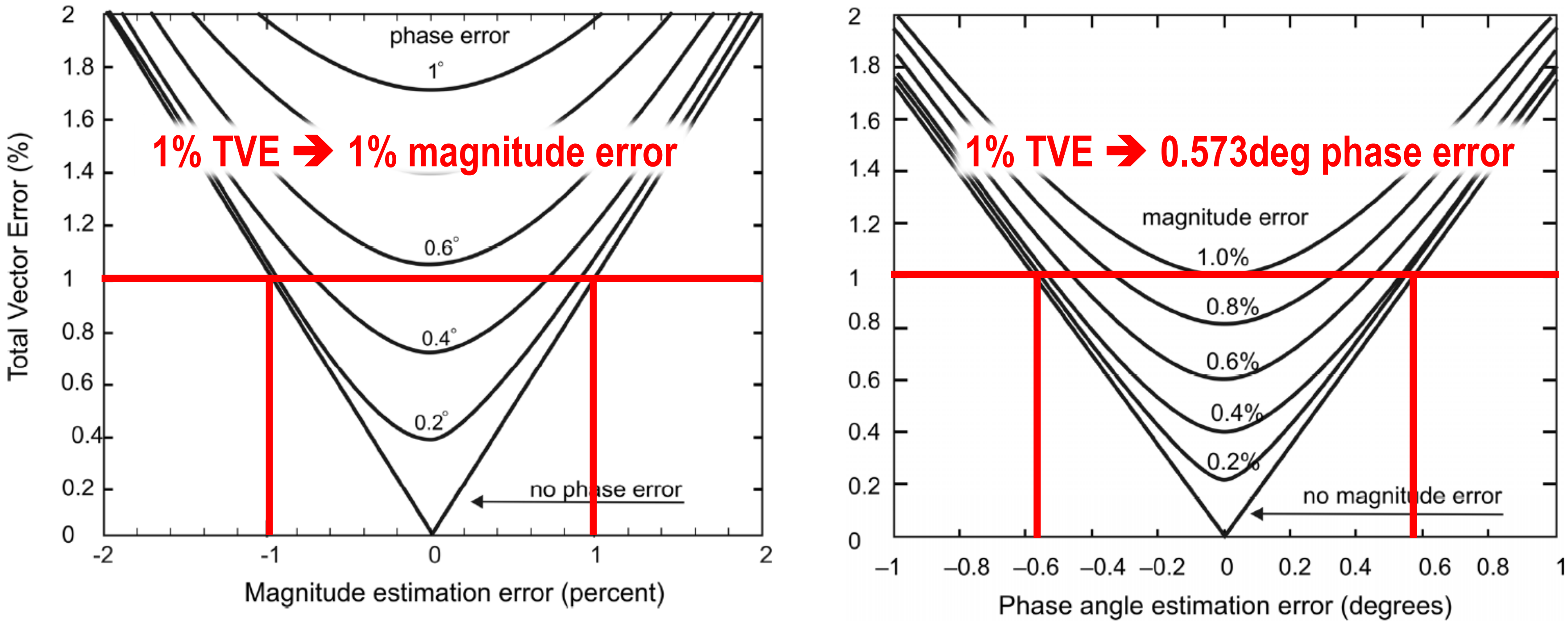
Reference (i.e. true) synchrophasor $X_r + jX_i$



Context – PMU Timing Module

Time errors and PMU accuracy

PMU accuracy metrics



Assuming **the error of a PMU entirely due to the phase** (i.e. the estimation of the phasor amplitude is perfect), we have that a TVE=1% corresponds to a phase error $\Delta\varphi = 0.573\text{deg} = 0.01 \text{ rad}$. For a power grid with a rated frequency of f_0=50Hz, this corresponds to a **timing error** Δt:

$$\Delta t = \frac{\Delta\varphi}{2\pi f_0} = \frac{0.01 rad}{2\pi \cdot 50 Hz} = 31.8 \cdot 10^{-6} s$$

Context – Time dissemination

Primary source for time → Time dissemination technology → End users

The final time accuracy and stability are due to the contribution of:

Performance of source

Performance of distribution

Performance of receiver

Outline

- **Primary sources for time**
- **Time dissemination technologies**
  - Satelite-based (eg: GPS)
  - Network-based (eg: PTP)
- **Synchronization performance metrics**
  - Precision, Stability and Accuracy

Primary sources for time

Introduction on oscillators

- Clock = Oscillator + Counter + Reference

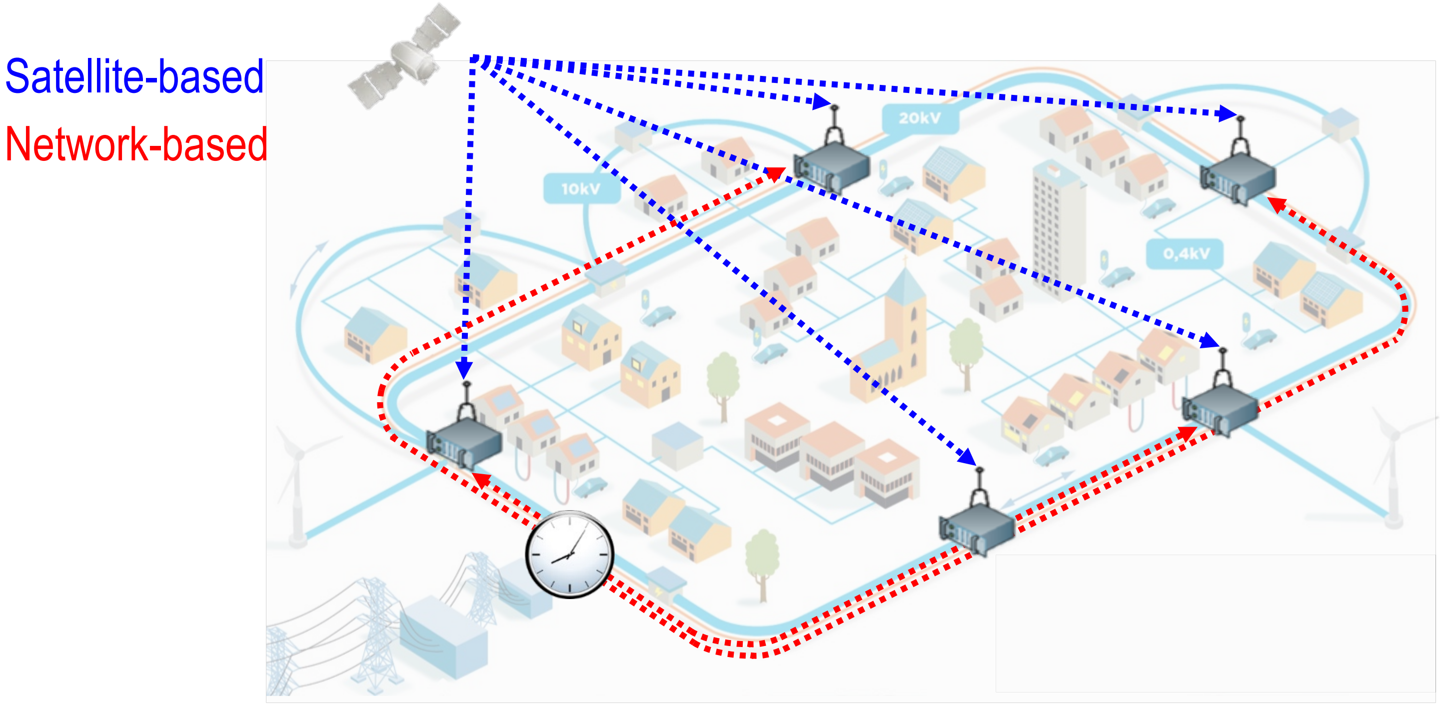
  - **Oscillator**: provides a signal of period T (with a corresponding frequency $\nu_0 = 1/T$), which is supposedly known (accurate) and stable ➔ e.g.: *earth, pendulum, spring, quartz, resonant circuit, etc*.

  - **Counter**: has the function to provide a direct link between those oscillations and an output needed by the user of the clock ➔ e.g.: *escapement, gear, dial, electronic frequency counter, etc*.

  - **Reference**: used in some clocks to reduce instabilities of the oscillator frequency by adjusting it so as it matches the reference frequency.

Time dissemination technologies

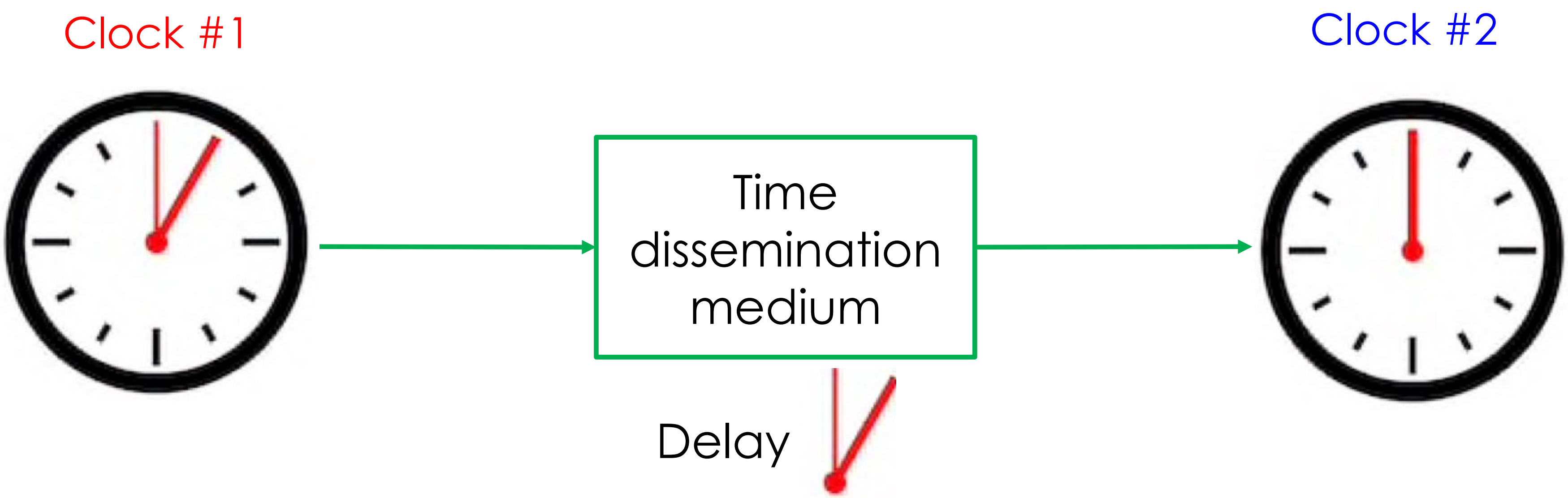
How to deliver a time reference?



- Accuracy and stability are the concerns
  - Time transfer accuracy ➔ Calibrating delays
  - Time transfer stability ➔ Keep the delay constant

Time dissemination technologies

One-Way comparison system



We would like to (continoulsy) align clock #2 to clock #1.
If the clock #1 sends its time to clock #2, this last will receive it with a delay that, a priori, is unknown. So, when clock #2 shows the time of clock #1, it will be with a lag corresponding to this delay.
Problem: how to measure this delay continoulsy such that clock #2 can be constantly aligned with clock #1 ?

Time dissemination technologies - GPS

The main approach used to solve the problem of the one-way comparison is the **use of multiple time sources (multiple masters) streaming the same time to a receiver**. This is how the **GPS** system works.

GPS constellation: satellites equipped with primary time sources. Each satellite streams messages containing its position (with respect to an arbitrary reference system) and absolute time.

Receiver: device capable to read messages from satellites and determine the receiver's position on the earth surface along with the absolute time.

Time dissemination technologies - GPS

Each receiver reads messages from a number of satellites (at least four are needed as we will clarify).

Each ith satellite **sends four pieces of information**:

- **Geocentric coordinates** $(x_i,\ y_i,\ z_i)$. These coordinates are expressed in meters from the center of the earth.
- **Absolute time at the transmission** of the message t_i.

The receiver gets this information at time T_i, measured on the receiver's clock.

Assuming the messages propagates at the speed of light c, we can express the distance between the receiver and the ith satellites with the so-called ***pseudorange***:

$$\rho_i = c\left(T_i - t_i\right)$$

Time dissemination technologies - GPS

To simplify the notation, the receiver get from the ith satellite the array:

$$\mathbf{s}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \\ \rho_i \end{bmatrix}$$

Problem: determine the geocentric coordinates of the receiver and adjust its clock.

Since **all the clocks of the GPS satellites are in synch each other**, **the receiver's clock will be out of synch with each satellite by the same amount**, say Δt. Then, the receiver's error in the pseudorange is $b=c\Delta t$ and each satellite's pseudorange ρ_i is is off of the same distance b.

Time dissemination technologies - GPS

Let indicate the actual receiver's position as (x, y, z). **The unknown receiver's quantities are:**

$$\mathbf{u} = \begin{bmatrix} x \\ y \\ z \\ b \end{bmatrix}$$

Problem constraint

The actual distance between the satellite and the receiver, plus the error, is equal to the pseudorange.

$$\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + b = \rho_i$$

$$(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 = (\rho_i - b)^2$$

To solve the above equations for $\mathbf{u}$, we need, at least, information from **four satellites** (4 unknowns).

Time dissemination technologies - GPS

In general, we would like to **solve the problem for a generic number of information collected by the receiver**. To do so, there is a **closed form** way to perform this calculus derived by Bancroft in 1985. Let expand the previous equation and re-arrange it.

$$x_i^2 - 2x_i x + x^2 + y_i^2 - 2y_i y + y^2 + z_i^2 - 2z_i z + z^2 = \rho_i^2 - 2\rho_i b + b^2$$

$$\left(x_i^2 + y_i^2 + z_i^2 - \rho_i^2\right) - 2\left(x_i x + y_i y + z_i z - \rho_i b\right) + \left(x^2 + y^2 + z^2 - b^2\right) = 0$$

Given the form of the variables within the parenthesis, we can introduce a **modified dot product called *Lorentz inner product*** defined as follows.

$$\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3 - u_4 v_4$$

Therefore, the problem constraint becomes the following.

$$\langle \mathbf{s}_i, \mathbf{s}_i \rangle - 2\langle \mathbf{s}_i, \mathbf{u} \rangle + \langle \mathbf{u}, \mathbf{u} \rangle = 0 \quad \text{or} \quad \frac{1}{2}\langle \mathbf{s}_i, \mathbf{s}_i \rangle - \langle \mathbf{s}_i, \mathbf{u} \rangle + \frac{1}{2}\langle \mathbf{u}, \mathbf{u} \rangle = 0$$

Time dissemination technologies - GPS

Let now write the problem in a matrix form assuming the receiver has information from n satellites ($n \geq 4$).

$$\mathbf{B} = \begin{bmatrix} x_1 & y_1 & z_1 & -\rho_1 \\ x_2 & y_2 & z_2 & -\rho_2 \\ x_3 & y_3 & z_3 & -\rho_3 \\ x_4 & y_4 & z_4 & -\rho_4 \\ \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & z_n & -\rho_n \end{bmatrix}, \quad \mathbf{a} = \frac{1}{2}\begin{bmatrix} \langle \mathbf{s}_1, \mathbf{s}_1 \rangle \\ \langle \mathbf{s}_2, \mathbf{s}_2 \rangle \\ \langle \mathbf{s}_3, \mathbf{s}_3 \rangle \\ \langle \mathbf{s}_4, \mathbf{s}_4 \rangle \\ \vdots \\ \langle \mathbf{s}_n, \mathbf{s}_n \rangle \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \text{ and } \Gamma = \frac{1}{2}\langle \mathbf{u}, \mathbf{u} \rangle$$

Using the above notation, the original problem constraint

$$\frac{1}{2}\langle \mathbf{s}_i, \mathbf{s}_i \rangle - \langle \mathbf{s}_i, \mathbf{u} \rangle + \frac{1}{2}\langle \mathbf{u}, \mathbf{u} \rangle = 0$$

can be written as follows.

$$\mathbf{a} - \mathbf{B}\mathbf{u} + \Gamma\mathbf{e} = 0 \quad \text{or} \quad \mathbf{B}\mathbf{u} = (\mathbf{a} + \Gamma\mathbf{e})$$

Time dissemination technologies - GPS

We can solve the problem by formulating it as a least square problem.

$$\mathbf{B}^T\mathbf{B}\mathbf{u} = \mathbf{B}^T(\mathbf{a} + \Gamma\mathbf{e}) \text{ and solve for } \mathbf{u}, \text{ say } \mathbf{u}^*$$

$$\mathbf{u}^* = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T(\mathbf{a} + \Gamma\mathbf{e}) = \mathbf{B}^+(\mathbf{a} + \Gamma\mathbf{e})$$

However, solving for $\mathbf{u}^*$ involves the knowledge of Γ that requires the unknown $\mathbf{u}$. Let replace the solution $\mathbf{u}^*$ into the Lorentz inner product and let us use its linearity property.

$$\Gamma = \frac{1}{2}\langle\mathbf{u}^*, \mathbf{u}^*\rangle = \frac{1}{2}\left\langle\mathbf{B}^+(\mathbf{a} + \Gamma\mathbf{e}), \mathbf{B}^+(\mathbf{a} + \Gamma\mathbf{e})\right\rangle = \frac{1}{2}\left\langle\mathbf{B}^+\mathbf{a}, \mathbf{B}^+\mathbf{a}\right\rangle + \Gamma\left\langle\mathbf{B}^+\mathbf{a}, \mathbf{B}^+\mathbf{e}\right\rangle + \frac{1}{2}\Gamma^2\left\langle\mathbf{B}^+\mathbf{e}, \mathbf{B}^+\mathbf{e}\right\rangle$$

$$\Gamma^2\left\langle\mathbf{B}^+\mathbf{e}, \mathbf{B}^+\mathbf{e}\right\rangle + 2\Gamma\left(\left\langle\mathbf{B}^+\mathbf{a}, \mathbf{B}^+\mathbf{e}\right\rangle - 1\right) + \left\langle\mathbf{B}^+\mathbf{a}, \mathbf{B}^+\mathbf{a}\right\rangle = 0$$

The previous equation is quadratic in Γ and all the coefficients can be computed since they do not depend on the unknowns.

Time dissemination technologies - GPS

To summarize, this is the Bancroft algorithm:

1. Organize data into the matrix/vectors $\mathbf{B}$, $\mathbf{a}$ and $\mathbf{e}$;
2. Solve the quadratic equation

$$\Gamma^2 \left\langle \mathbf{B}^+\mathbf{e}, \mathbf{B}^+\mathbf{e} \right\rangle + 2\Gamma \left(\left\langle \mathbf{B}^+\mathbf{a}, \mathbf{B}^+\mathbf{e} \right\rangle - 1 \right) + \left\langle \mathbf{B}^+\mathbf{a}, \mathbf{B}^+\mathbf{a} \right\rangle = 0$$

and determine the two roots Γ_1 and Γ_2.

3. Solve for the least-square solutions $\mathbf{u}^*(\Gamma_1)$ and $\mathbf{u}^*(\Gamma_2)$

$$\mathbf{u}^* = \left(\mathbf{B}^T \mathbf{B} \right)^{-1} \mathbf{B}^T \left(\mathbf{a} + \Gamma \mathbf{e} \right) = \mathbf{B}^+ \left(\mathbf{a} + \Gamma \mathbf{e} \right)$$

and pick the only $\mathbf{u}^*$ that provides a distance from the earth center that is compatible with the earth diameter.

4. Determine the receiver time synch error $\Delta t = b/c$ and synchronise its clock with the GPS.

Time dissemination technologies - GPS

Relativistic corrections

Consider a clock aboard a GPS satellite orbiting the Earth. There are two major relativistic influences upon its rate of timekeeping:

- **special relativistic correction** for its **orbital speed;**
- **general relativistic correction** for its **orbital altitude**.

Let's start with the **special relativistic correction**: from an earthbound receiver, the transmitting clock is subject to time dilation due to its **orbital speed**. A clock aboard a GPS satellite traveling at speed v runs slow (compared to a stationary clock) by a factor of

$$\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{v^2}{2c^2} \quad (since\ v \ll c)$$

Time dissemination technologies - GPS

Relativistic corrections

Therefore, when one second of time elapses on the Earth, the moving clock loses $v^2/2c^2 = K/E_0$ seconds, where K and E_0 are the kinetic and rest energies of the clock, respectively.

Time dissemination technologies - GPS

Relativistic corrections

Regarding the **general relativistic correction**, a clock at the higher gravitational potential of orbit **runs faster than a surface clock**. The **gravitational potential energy** of a body of mass m at distance r from the center of the Earth of mass m_E, in Earth's gravity is $U = mV$, where $V = -Gm_E/r$ is Earth's gravitational potential and G is the gravitational constant.

In the case of a photon, we replace m by hf/c^2, where hf is the **photon's energy** being f its frequency and h the Planck's constant.

If the **photon travels downward in Earth's gravitational field, it loses potential energy** equal to $(hf/c^2)\Delta V$ and gains an equal amount of kinetic energy $h\Delta f$.

Time dissemination technologies - GPS

Relativistic corrections

We thereby deduce that the falling photon is gravitationally blue-shifted by

$$\Delta f = f\,\frac{\Delta V}{c^2}$$

In other words, **if the clock's ticking is synchronized to a light wave, the orbiting clock will be observed at Earth's surface to be ticking faster due to this gravitational frequency shift.**

Therefore, when one second of Earth time elapses, the clock at high altitude gains $\Delta V/c^2 = \Delta U/E_0$ seconds, where U is the gravitational potential energy of the clock.

Time dissemination technologies - GPS

Relativistic corrections

The sum of the two relativistic effects can be expressed as:

$$\frac{\Delta t}{\tau} = \frac{K - U}{E_0}$$

where Δt is the time lost by the orbiting clock when a time interval τ elapses on the surface-bound clock and $K - U$ is the Lagrangian of the orbiting clock where the reference level for the gravitational potential energy is chosen to lie at Earth's surface.

Time dissemination technologies - GPS

Relativistic corrections

Example: let's calculate the amount of these two effects for a GPS satellite, located at an altitude of $r = 26580\ km$ (i.e., about 4x Earth's radius of $r_E = 6380\ km$).

From Newton's law, we have:

$$a = \frac{F}{m} \rightarrow \frac{v^2}{r} = \frac{Gm_E}{r^2} \rightarrow v^2 = \frac{gr_E^2}{r}$$

where $g = Gm_E/r_E^2 = 9.8\ m/s^2$ is the Earth's surface gravitational acceleration. Therefore, the **fractional time loss due to the satellite's orbital speed is**

$$-\frac{v^2}{2c^2} = -\frac{gr_E^2}{2rc^2} = -8.3 \cdot 10^{-11} \rightarrow -7.2\ \mu s/day$$

Relativistic corrections

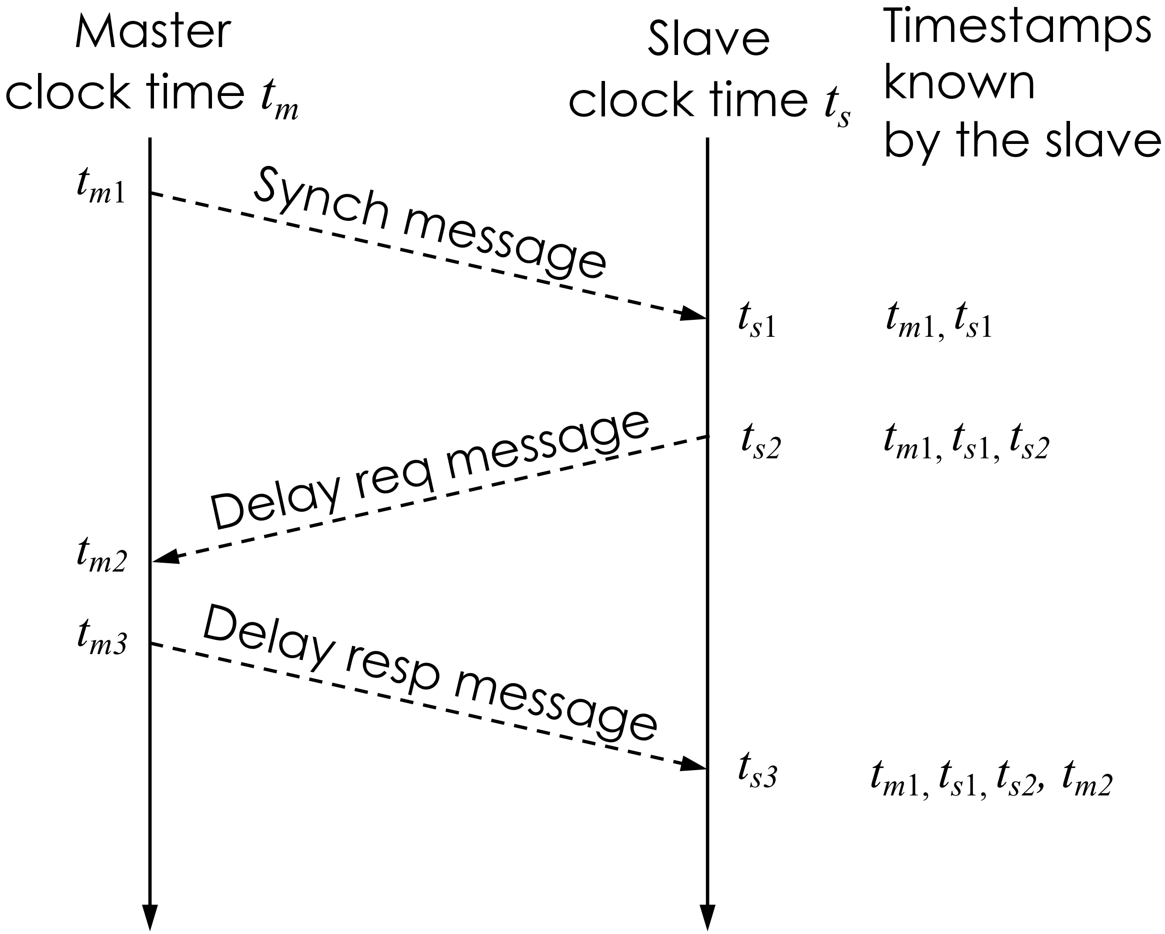
Example: regarding the second factor related to the **general relativistic correction for the satellite's orbital altitude it is**:

$$\frac{\Delta V}{c^2} = \frac{1}{c^2}\left(-\frac{Gm_E}{r} + \frac{Gm_E}{r_E}\right) = \frac{gr_E}{c^2}\left(1 - \frac{r_E}{r}\right) = +5.28 \cdot 10^{-10}$$
$$\rightarrow +45.6\ \mu s/day$$

It is interesting to note that the general relativistic correction for the satellite's orbital altitude is six times larger than the fractional time loss due to the satellite's orbital speed.

These two effects are embedded in the messages sent by the satellites to the reveivers.

Two-Way comparison system (eg. PTP)

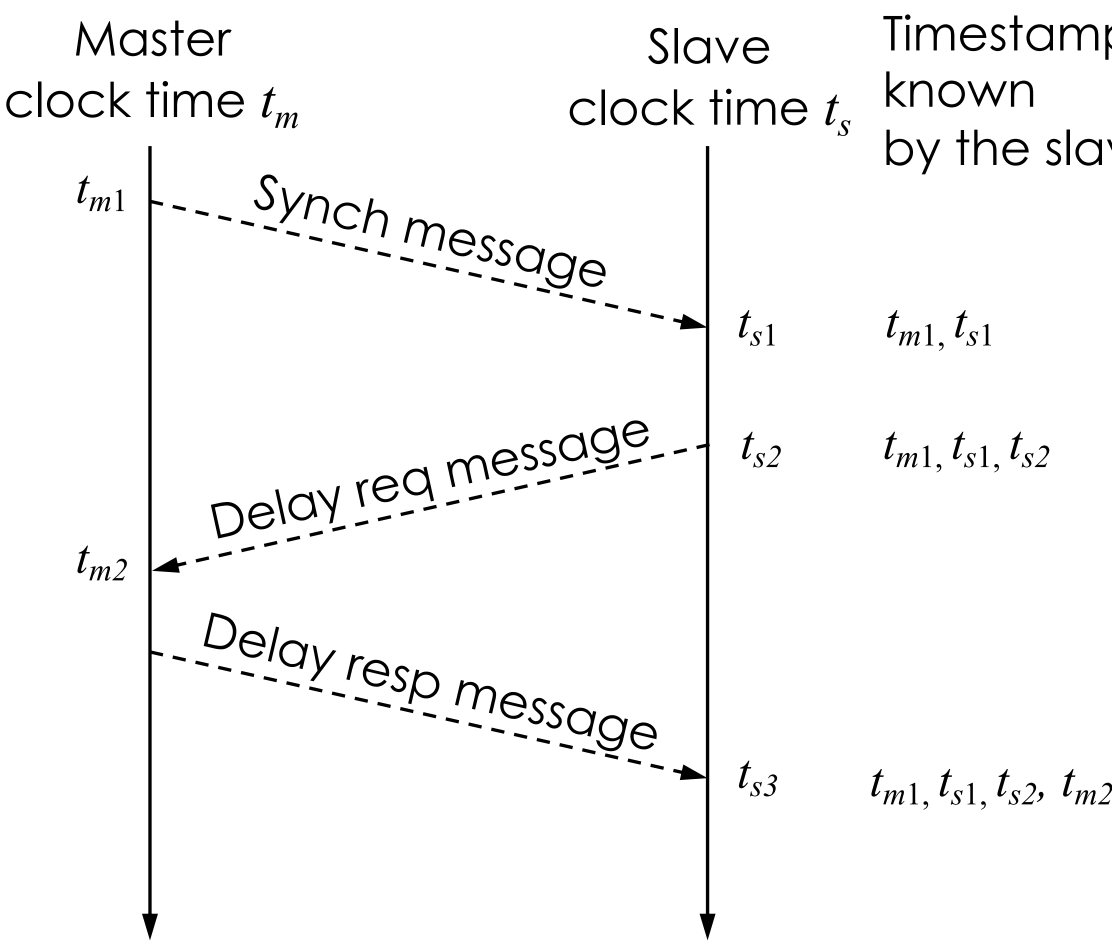


For a given **slave clock**, we are interested to compute the **offset** o at a **given time** t, namely, $o(t)$ defined as:

$$o(t) = t_s(t) - t_m(t)$$

At t_{m1}, the master clock sends a **synch message** to the slave who receives this information at t_{s1}. Then, the slave knows both t_{m1} and t_{s1}. Then, the slave sends a **delay request message** to the master at t_{s2}. The message is received by the master at t_{m2}. The master replies with a **delay response message** that, at t_{s3}, is received and the slave knows: $t_{s1}, t_{s2}, t_{m1}, t_{m2}$

Two-Way comparison system (eg. PTP)



If the two clocks have an **offset time, say** o, and the **propagation delay is** d, we have:

$$t_{s1} - t_{m1} = o + d$$

$$t_{m2} - t_{s2} = -o + d$$

Therefore, **if the delay** d **is the same (symmetric) for all the messages,** the slave can compute the offset and align its clock to the master:

$$o = \frac{(t_{s1} - t_{m1}) - (t_{m2} - t_{s2})}{2}$$

$$d = \frac{(t_{s1} - t_{m1}) + (t_{m2} - t_{s2})}{2}$$

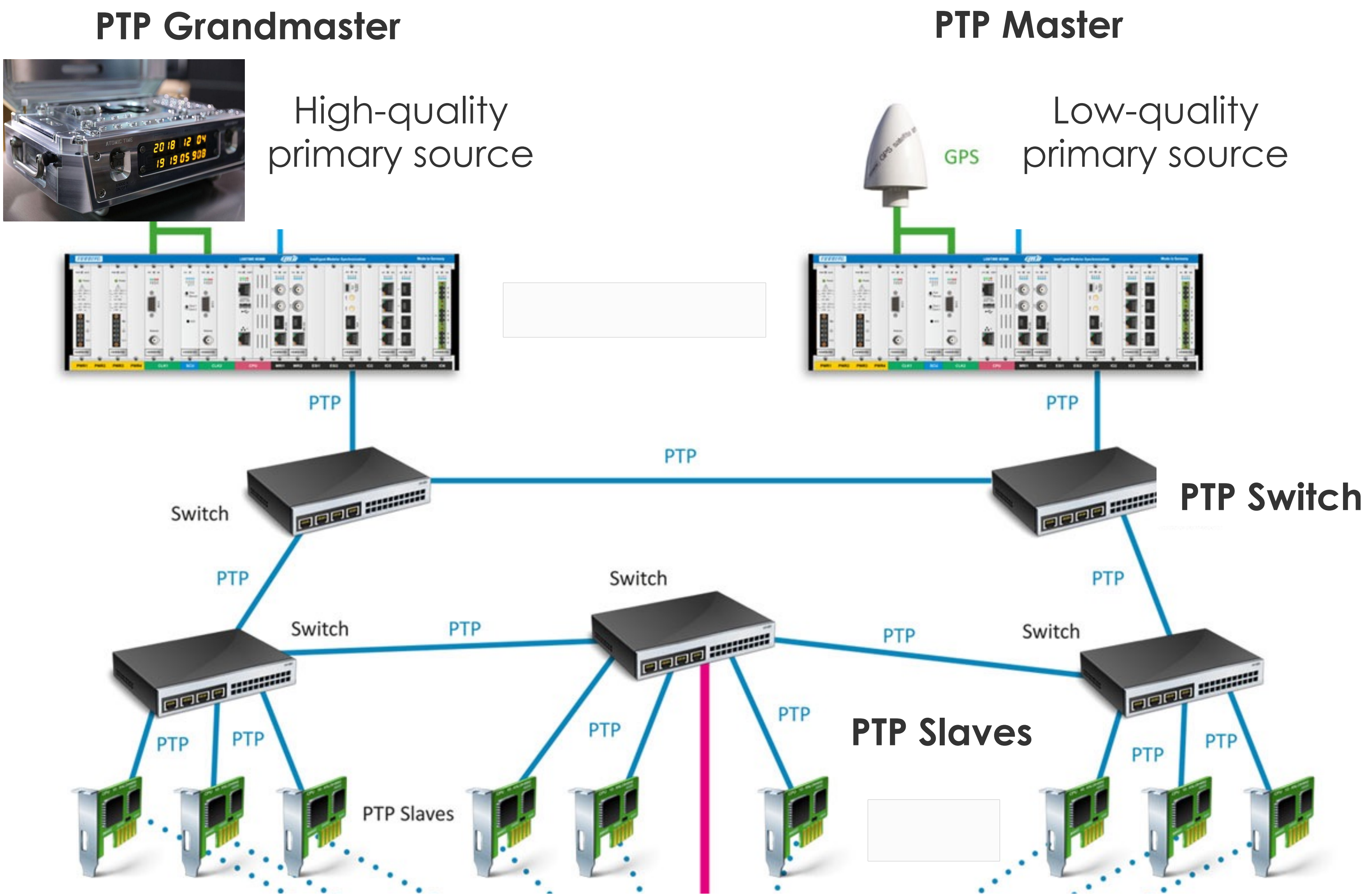
Time dissemination technologies

Time synchronization techniques for PMUs

<table>
<tr><th></th><th>Satellite-based</th><th colspan="2">Network-based</th></tr>
<tr><td></td><td>Global Position System (GPS)</td><td>Precision Time Protocol (PTP)</td><td>White Rabbit (WR)</td></tr>
<tr><td>Accuracy</td><td>100 ns</td><td>1 µs</td><td>1 ns</td></tr>
<tr><td rowspan="2">Features</td><td rowspan="2">▪ Low installation cost
▪ Widely used</td><td colspan="2">▪ Time dissemination relies on the same Ethernet physical layer as PMU data transfer</td></tr>
<tr><td>▪ Standard profile for power systems</td><td>▪ Reliability
▪ Determinism</td></tr>
<tr><td>Limitations</td><td>▪ Accessibility
▪ Security</td><td>▪ Accuracy
▪ Non-determinism</td><td>▪ Fiber physical layer</td></tr>
</table>

Time dissemination technologies

Packed-Switching Synchronization Messaging Protocols



Synchronization performance metrics

- **Ideal oscillator** ➔ Absolutely accurate and stable sinus with amplitude A and frequency ν_0

$$s(t) = A\cos(2\pi\nu_0 t)$$

- **Real oscillator** ➔ The amplitude and the frequency of the output fluctuate

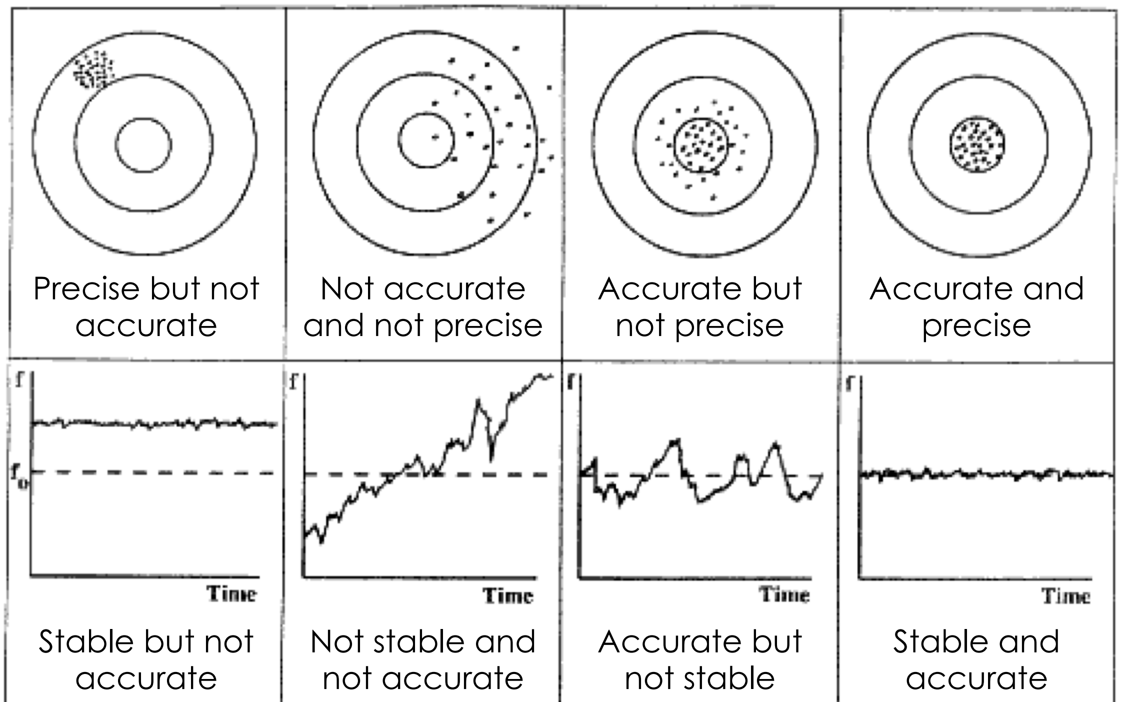
$$\tilde{s}(t) = A\big(1 + \varepsilon(t)\big)\cos\big(2\pi\nu_0 t + \Phi(t)\big)$$

- Time error: $x(t) = \frac{\Phi(t)}{2\pi\nu_0}$
- Normalized frequency error: $y(t) = \frac{dx(t)}{dt} = \frac{1}{2\pi\nu_0}\frac{d\Phi(t)}{dt} = \frac{\partial\nu(t)}{\nu_0}$
- Measured frequency: $\nu(t) = \nu_0 + \partial\nu(t)$
- $\partial\nu(t) = \nu_0 y(t) = \frac{1}{2\pi}\frac{d\Phi(t)}{dt}$
- In general $\lim_{T\to\infty}\frac{1}{T}\int_t^{t+T}\partial\nu(t)dt$ diverges, $\partial\nu(t)$ = offset + random

Synchronization performance metrics

Accuracy and stability

- The clock **accuracy describes how well the actual frequency matches the specified frequency**. Clock accuracy might be affected by factors such as the quality of the oscillator crystal and how the oscillator was assembled.
- The clock **stability describes how well the oscillator frequency resists fluctuations**. The dominant factor that affects stability is a variation in temperature, though aging over time, supply voltage, shock, vibration, and capacitive load that the clock must drive can all affect the clock stability.



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